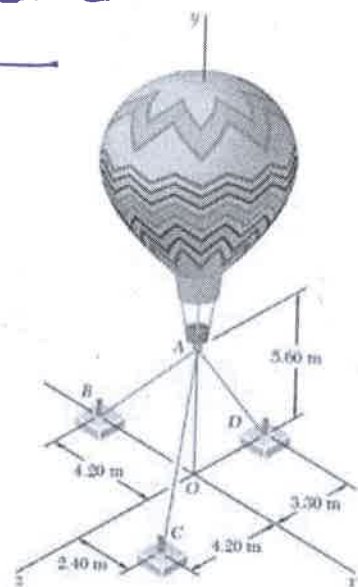


# Solution of Assignment #3

## Moment of Equilibrium

(1)

- (1) ∴ the coordinates of Point A = (0, 5.6, 0)  
 ~ ~ ~ ~ B = (-4.2, 0, 0)  
 ~ ~ ~ ~ C = (2.4, 0, 4.2)  
 ~ ~ ~ ~ D = (0, 0, -3.3)



∴ the force  $\vec{P} = P\hat{j}$

∴ the force @ Cable AB is  $\vec{T}_{AB} = T_{AB} \cdot \hat{e}_{AB}$

$$\therefore \vec{T}_{AB} = T_{AB} \cdot \frac{\vec{AB}}{|\vec{AB}|} = T_{AB} \cdot \frac{-4.2\hat{i} - 5.6\hat{j}}{\sqrt{(4.2)^2 + (5.6)^2}} = \frac{T_{AB}}{7} (-4.2\hat{i} - 5.6\hat{j})$$

∴ the force @ Cable AC is  $\vec{T}_{AC} = T_{AC} \cdot \hat{e}_{AC}$

$$\therefore \vec{T}_{AC} = T_{AC} \cdot \frac{\vec{AC}}{|\vec{AC}|} = T_{AC} \cdot \frac{2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k}}{\sqrt{(2.4)^2 + (5.6)^2 + (4.2)^2}} = \frac{T_{AC}}{7.4} (2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k})$$

∴ the force @ Cable AD is  $\vec{T}_{AD} = T_{AD} \cdot \hat{e}_{AD}$

$$\therefore \vec{T}_{AD} = T_{AD} \cdot \frac{\vec{AD}}{|\vec{AD}|} = T_{AD} \cdot \frac{-5.6\hat{j} - 3.3\hat{k}}{\sqrt{(5.6)^2 + (3.3)^2}} = \frac{T_{AD}}{6.5} (-5.6\hat{j} - 3.3\hat{k})$$

And ∴ The point (A) is in equilibrium state then,

$$\sum f_x = \text{zero} ; -\frac{4.2}{7} T_{AB} + \frac{2.4}{7.4} T_{AC} = 0 \quad \therefore \underline{T_{AB} = 259 \text{ N "Given"}}$$

$$\therefore \underline{T_{AC} = 479.15 \text{ N}}$$

$$\sum f_y = \text{zero} ; P - \frac{5.6}{7} T_{AB} - \frac{5.6}{7.4} T_{AC} - \frac{5.6}{6.5} T_{AD} = 0 \quad \rightarrow \text{(I)}$$

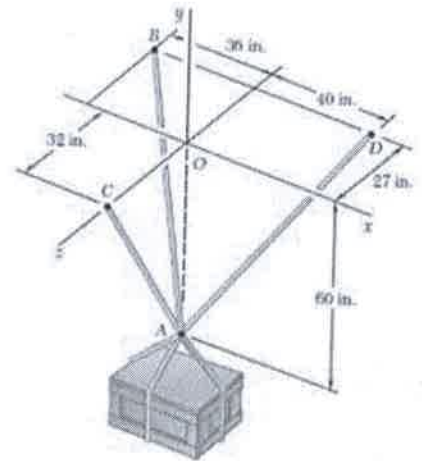
$$\sum f_z = \text{zero} ; \frac{4.2}{7.4} T_{AC} - \frac{3.3}{6.5} T_{AD} = 0$$

$$\therefore \text{from Equ. (I)} \quad \underline{P = 1031.29 \text{ N}} \quad \therefore \underline{T_{AD} = 535.66 \text{ N}}$$

(2)

[2] Given: Each Cable Can Sustain a max. Tension = 616-lb.

Req.: the maximum weight of the Gate.



Solution:

∴ The Coordinates of Point A = (0, -60, 0)  
 " " " " B = (-36, 0, -27)  
 " " " " C = (0, 0, 32)  
 " " " " D = (40, 0, -27)

In this problem we have 4-unknowns (the weight (w) and Tension force in 3-Cables)

first: Get the direction of each force.

∴  $\vec{w} = -w\vec{j}$

∴ The force @ cable AB is  $\vec{T}_{AB} = T_{AB} \cdot \hat{e}_{AB}$

∴  $\vec{T}_{AB} = T_{AB} \cdot \frac{-36\vec{i} + 60\vec{j} - 27\vec{k}}{\sqrt{(36)^2 + (60)^2 + (27)^2}} = \frac{T_{AB}}{75} (-36\vec{i} + 60\vec{j} - 27\vec{k})$

∴ The force @ cable AC is  $\vec{T}_{AC} = T_{AC} \cdot \hat{e}_{AC}$

∴  $\vec{T}_{AC} = T_{AC} \cdot \frac{0\vec{i} + 60\vec{j} + 32\vec{k}}{\sqrt{(60)^2 + (32)^2}} = \frac{T_{AC}}{68} (60\vec{j} + 32\vec{k})$

∴ The force @ cable AD is  $\vec{T}_{AD} = T_{AD} \cdot \hat{e}_{AD}$

∴  $\vec{T}_{AD} = T_{AD} \cdot \frac{40\vec{i} + 60\vec{j} - 27\vec{k}}{\sqrt{(40)^2 + (60)^2 + (27)^2}} = \frac{T_{AD}}{77} (40\vec{i} + 60\vec{j} - 27\vec{k})$

Second: Put the equilibrium Equations.

$\sum F_x = 0$  ∴  $-\frac{36}{75} T_{AB} + \frac{40}{77} T_{AD} = 0 \rightarrow \text{I}$

$\sum F_y = 0$  ∴  $-w + \frac{60}{75} T_{AB} + \frac{60}{68} T_{AC} + \frac{60}{77} T_{AD} = 0 \rightarrow \text{II}$

$\sum F_z = 0$  ∴  $-\frac{27}{75} T_{AB} + \frac{32}{68} T_{AC} - \frac{27}{77} T_{AD} = 0 \rightarrow \text{III}$

(3)

So, we get 3-equations with 4-unknowns.

∴ we must assume one unknown to obtain others

∴ Assume that the tension at cable  $\overline{AC} \Rightarrow T_{AC} = 616 \text{ lb}$

from Equ. (III)  $\frac{27}{75} T_{AB} + \frac{27}{77} T_{AD} = \frac{32}{68} * 616 = 289.882 \rightarrow \text{(III')}$

Equ. (I)  $-\frac{36}{75} T_{AB} + \frac{40}{77} T_{AD} = 0 \rightarrow \text{(I')}$

∴  $\frac{2052}{77} T_{AD} = 289.882 * 36 \Rightarrow \boxed{\therefore T_{AD} = 391.595 \text{ lb}}$

∴ from Equ. (I)  $\boxed{T_{AB} = 423.8 \text{ lb}}$   
 $\boxed{T_{AB} < 616 \text{ lb ok.}}$

∴ from Equ. (II) we can get the max. weight of the Gate

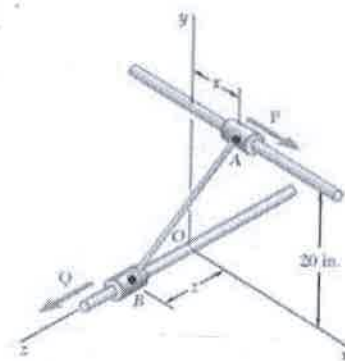
∴  $W = \frac{60}{75} * 423.8 + \frac{60}{68} * 616 + \frac{60}{77} * 391.595$

$\boxed{\therefore W = 1187.71 \text{ lb}}$

(4)

3 Given: wire length = 25-in  
 force  $Q = 60\text{-lb}$   
 $x = 9\text{-in}$

Req.: (a) Tension in wire  
 (b) the force (P)



Solution :

∴ the Co-ordinates of Point A = (9, 20, 0)

∴ the Co-ordinates of Point B = (0, 0, z)

$$\therefore \vec{BA} = A - B = 9\vec{i} + 20\vec{j} - z\vec{k}$$

∴ the magnitude of the vector  $|\vec{BA}|$  = the length of wire

$$\therefore |\vec{BA}| = \sqrt{(9)^2 + (20)^2 + (z)^2} = 25 \implies \underline{\underline{z = 12\text{-in}}}$$

$$\therefore \vec{BA} = 9\vec{i} + 20\vec{j} - 12\vec{k}$$

∴ The point (B) is in Equilibrium State

$$\therefore \text{the tension in wire } \vec{T} = T * \hat{e}_{BA} = \frac{T}{25} * (9\vec{i} + 20\vec{j} - 12\vec{k})$$

$$\therefore \vec{Q} = 60\vec{k} = \left(\frac{T}{25} * 12\right)\vec{k}$$

$$\therefore \boxed{T = 125\text{-lb}}$$

∴ The point (A) is in Equilibrium State

$$\therefore \vec{P} = P\vec{i} = \left(\frac{T}{25} * 9\right)\vec{i}$$

$$\therefore \boxed{P = 45\text{-lb}}$$

(5)

[4] Given: Tension in Cable  $\overline{BH} = 450 \text{ N}$

Req.: Moment about line  $\overline{AD}$

Solution:

∴ The moment about line equal the moment about point in the line multiplied by the unit vector of the line.

$$\therefore M_{\overline{AD}} = \vec{M}_A \cdot \hat{e}_{AD}$$

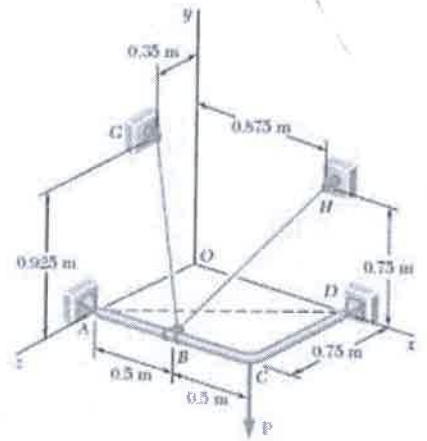
$$\therefore \text{The moment } \vec{M}_A = \overrightarrow{AB} \times \vec{F}_{BH}$$

∴ The Co-ordinates of Point  $A = (0, 0, 0.75)$

" " " "  $B = (0.5, 0, 0.75)$

" " " "  $D = (1.0, 0, 0)$

" " " "  $H = (0.875, 0.75, 0)$



$$\therefore \overrightarrow{AB} = B - A = 0.5 \hat{i}$$

$$\therefore \vec{F}_{BH} = F \cdot \hat{e}_{BH} = 450 \cdot \frac{0.375 \hat{i} + 0.75 \hat{j} - 0.75 \hat{k}}{\sqrt{(0.375)^2 + (0.75)^2} \cdot 2} = 450 \cdot (0.375, 0.75, -0.75)$$

$$\therefore \vec{F}_{BH} = 150 \hat{i} + 300 \hat{j} - 300 \hat{k}$$

$$\therefore \vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix} = \hat{i}(0) + \hat{j}(150) + \hat{k}(150)$$

$$\therefore \hat{e}_{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{1.0 \hat{i} - 0.75 \hat{k}}{\sqrt{(1)^2 + (0.75)^2}} = 0.8 \hat{i} - 0.6 \hat{k}$$

$$\therefore M_{\overline{AD}} = \vec{M}_A \cdot \hat{e}_{AD} = (150 \hat{j} + 150 \hat{k}) \cdot (0.8 \hat{i} - 0.6 \hat{k}) = -150 \cdot 0.6$$

$$\therefore M_{\overline{AD}} = -90 \text{ N}\cdot\text{m}$$